YILDIZ TECHNICAL UNIVERSITY
MECHANICAL FACULTY / MECHATRONICS ENGINEERING DEPARTMENT

Student Name:	**Number:**	**Signature:**
Course: MKT3841 Numerical Methods Gr2	**Date:** 12.01.2022 **Time:** 8:30	
Exam Type: Final Exam	**Course Instructor:** Assoc.Prof. Cüneyt Yılmaz	

Question 1) [33 Points] Obtain the finite difference equations in the matrix form for the following differential equation from x=0.0 to 0.8 for n=5:

$$\frac{d^2y}{dx^2} = -\frac{dy}{dx} + 5y$$

where y'(0.0) = 1 and y(0.8) = 5 .

	$f(x-2h)$	$f(x-h)$	$f(x)$	$f(x+h)$	$f(x+2h)$
$2hf'(x)$		-1	0	1	
$h^2 f''(x)$		1	-2	1	
$2h^3 f'''(x)$	-1	2	0	-2	1
$h^4 f^{(4)}(x)$	1	-4	6	-4	1

Solution 1) h=(0.8-0)/(n-1)= (0.8)/4 = 0.2

From the table: $y_i' = [1/(2h)](y_{i+1} - y_{i-1})$ and $y_i'' = (1/h^2)(y_{i-1} - 2y_i + y_{i+1})$

$y_i'' = -y_i' + 5y_i$

$(1/h^2)(y_{i-1} - 2y_i + y_{i+1}) = -[1/(2h)](y_{i+1} - y_{i-1}) + 5y_i$

$y_{i-1} - 2y_i + y_{i+1} = -(h/2)(y_{i+1} - y_{i-1}) + 5h^2 y_i$

$(1 - h/2)y_{i-1} - (2 + 5h^2)y_i + (1+h/2)y_{i+1} = 0$

$[1 - (0.2)/2]y_{i-1} - [2 + 5(0.2)^2]y_i + [1 + (0.2)/2]y_{i+1} = 0$

$0.9y_{i-1} - 2.2y_i + 1.1y_{i+1} = 0 \qquad i=1 \text{ to } 4$

$i=1: \quad 0.9y_0 - 2.2y_1 + 1.1y_2 = 0$

$y'_1 = 1 \rightarrow y'_1 = (y_2 - y_0)/(2h) = (y_2 - y_0)/0.4 = 1 \rightarrow y_0 = y_2 - 0.4$

$0.9(y_2 - 0.4) - 2.2y_1 + 1.1y_2 = 0$

$-2.2y_1 + 2y_2 = 0.36$

$i=2: \quad 0.9y_1 - 2.2y_2 + 1.1y_3 = 0$

$i=3: \quad 0.9y_2 - 2.2y_3 + 1.1y_4 = 0$

$i=4: \quad 0.9y_3 - 2.2y_4 + 1.1y_5 = 0$

$i=5: \quad y_5 = 5$

In the matrix form:

$$\begin{bmatrix} -2.2 & 2 & 0 & 0 & 0 \\ 0.9 & -2.2 & 1.1 & 0 & 0 \\ 0 & 0.9 & -2.2 & 1.1 & 0 \\ 0 & 0 & 0.9 & -2.2 & 1.1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 0.36 \\ 0 \\ 0 \\ 0 \\ 5 \end{bmatrix}$$

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Question 2) [33 Points] Solve the simultaneous equation system below by using **Gauss Elimination Method** with all details (clearly showing forward and backward calculations).

$$(3\,A)\,x_1 + (6\,A)\,x_2 + (15\,A)\,x_3 - (150\,A) = 0$$
$$(-B-1)\,x_1 + (-3\,B-3)\,x_2 + (2\,B+2)\,x_3 - (3\,B+3) = 0$$
$$(D+1)\,x_1 + (-D-1)\,x_2 + (C+1)\,x_3 - (8\,C+D+9) = 0$$

where AB and CD are the **first 2** and **last 2 digits of your school numbe**r, respectively.
Attention: Please pay your maximum attention when preparing your customized simultaneous equation set according to your school number: $A\,B\,\#\,\#\,\#\,\#\,C\,D$. No Small Street Market Calculation.

Solution 2) As one example:

$AB\,CD =$ 1 8 1 5

Prepare the simultaneous equation system:

3	x_1 +	6	x_2 +	15	x_3 =	150
-9	x_1 +	-27	x_2 +	18	x_3 =	27
6	x_1 +	-6	x_2 +	2	x_3 =	22

Gauss Elimination

3 x Row1 + Row2 → Row2:

3	x_1 +	6	x_2 +	15	x_3 =	150
0	x_1 +	**-9**	x_2 +	**63**	x_3 =	**477**
6	x_1 +	-6	x_2 +	2	x_3 =	22

-2 x Row1 + Row3 → Row3:

3	x_1 +	6	x_2 +	15	x_3 =	150
0	x_1 +	-9	x_2 +	63	x_3 =	477
0	x_1 +	**-18**	x_2 +	**-28**	x_3 =	**-278**

-2 x Row2 + Row3 → Row3:

3	x_1 +	6	x_2 +	15	x_3 =	150
0	x_1 +	-9	x_2 +	63	x_3 =	477
0	x_1 +	0	x_2 +	**-154**	x_3 =	**-1232**

Backward calculation:

$x_3 =$ -1232 / -154 → $x_3 = 8$

$x_2 =$ (477 - 63 * 8)/ -9 → $x_2 = 3$

$x_1 =$ (150 - 6 * 3 - 15 * 8)/ 3 → $x_1 = 4$

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Question 3) [34 Points] A car has a variable speed (in meter/seconds) with respect to time t (seconds) as $v(t) = 2 + 25\,t - 3t^2$. By using **Golden Bracketing** method, numerically calculate the maximum speed of this car. In your solution, apply the followings:

- Write the function to optimize and also add a constraint function considering a positive t value. Assume $\lambda=1$.
- Search the optimum time *t* between *3* and *4 seconds*.
- Stop telescoping when the bracketing width is less then $\varepsilon=0.3$.
- Use the Golden Ratio *R=0.618.*
- Work with at least 3 decimal points.

Solution 3) Prepare the function to do optimization:

To convert the problem from maximization to minimization, we negate the given function v(t). We also add the constraint function $[\min(0,t)]^2$ to ensure positive t values.

$$f(t) = -2 - 25\,t + 3\,t^2 + [\min(0,t]^2$$

and numerically minimize it by using Golden Bracketing method as:

i	a_i	b_i	h	$x_1=b_i-R_xh$	$x_2=a_i+R_xh$	$f(x_1)$	$f(x_2)$	Stop if $h<\varepsilon$
0	3.000	4.000	1.000	3.382	3.618	-52.236	-53.180	
1	3.382	4.000	0.618	3.618	3.764	-53.180	-53.597	
2	3.618	4.000	0.382	3.764	3.854	-53.597	-53.790	
3	3.764	4.000	0.236	3.854	3.910	-53.790	-53.885	YES

$$t_{max} = \frac{3.764 + 4.000}{2} = 3.882\ s \qquad v_{max} = 53.840\ m/s$$

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