Yıldız Technical University, Department of Mechatronics Engineering
Numerical Methods MKT4712, Final Makeup Exam , June 24, 2016

Question 4) Consider the following 4th order differential equation:
$y^{(4)}=y''+y$, with the boundary conditions, $y(0) = 0$, $y'(0) = 1$, $y(3) = 5$, $y''(3) = 0$.
Obtain the solution as a simultaneous equation **A·y=b** by using the central finite difference approximation.
Use $\Delta x=1$.

	$f(x-2h)$	$f(x-h)$	$f(x)$	$f(x+h)$	$f(x+2h)$
$2hf'(x)$		-1	0	1	
$h^2 f''(x)$		1	-2	1	
$2h^3 f'''(x)$	-1	2	0	-2	1
$h^4 f^{(4)}(x)$	1	-4	6	-4	1

Solution 4) $n = \frac{3-0}{1} + 1 = 4$

From the table:

$$y_i^{(4)} = \frac{1}{h^4}\left(y_{i-2} - 4y_{i-1} + 6y_i - 4y_{i+1} + y_{i+2}\right)$$

$$y_i' = \frac{1}{2h}(y_{i+1} - y_{i-1})$$

$$y_i'' = \frac{1}{h^2}(y_{i-1} - 2y_i + y_{i+1})$$

Substituting these into the differential equation: $y^{(4)} - y'' - y = 0$

$$y_{i-2} - 4y_{i-1} + 6y_i - 4y_{i+1} + y_{i+2} - y_{i-1} + 2y_i - y_{i+1} - y_i = 0$$

$$y_{i-2} - 5y_{i-1} + 7y_i - 5y_{i+1} + y_{i+2} = 0$$

$i=1$: $\boxed{y_1 = 0}$

$i=2$: $y_0 - 5y_1 + 7y_2 - 5y_3 + y_4 = 0$

$y_1' = 1 \Rightarrow \frac{1}{2}(y_2 - y_0) = 1 \Rightarrow y_0 = y_2 - 2$

$\Rightarrow y_2 - 2 - 5y_1 + 7y_2 - 5y_3 + y_4 = 0$

$$\boxed{-5y_1 + 8y_2 - 5y_3 + y_4 = 2}$$

$i=3$: $y_1 - 5y_2 + 7y_3 - 5y_4 + y_5 = 0$

$y_4'' = y_3 - 2y_4 + y_5 = 0 \Rightarrow y_5 = 2y_4 - y_3$

$\Rightarrow y_1 - 5y_2 + 7y_3 - 5y_4 + 2y_4 - y_3 = 0$

$$\boxed{y_1 - 5y_2 + 6y_3 - 3y_4 = 0}$$

$i=4$: $\boxed{y_4 = 5}$

In the matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -5 & 8 & -5 & 1 \\ 1 & -5 & 6 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 5 \end{bmatrix}$$

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