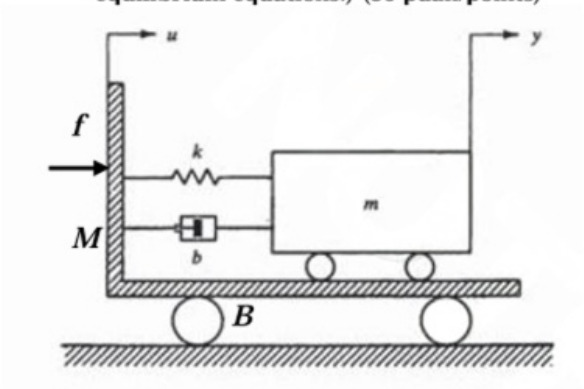
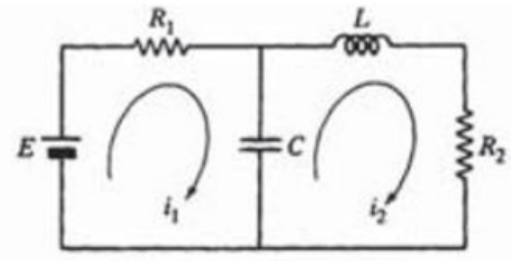
YILDIZ TECHNICAL UNIVERSITY
MECHATRONICS ENGINEERING DEPARTMENT
MKT3801 202201 VİZE/MIDTERM

Gr….. Adı/Name: Soyadı/Surname: #:

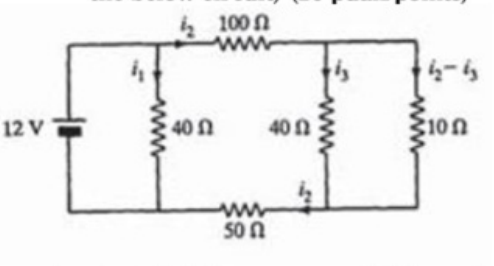
1. Aşağıdaki c. **iki kütleli mekanik sistemin hareketinin diferansiyel denklemi**ni çıkarınız. *f,* dış kuvveti, *B,* yer ile *M* kütlesi arasındaki viskoz sürtünmeyi, *M* de, arabanın kütlesini temsil etmektedir. a. **Serbest cisim diagramı**nı çiziniz ve b. **kuvvet denklemleri**ni yazınız. (c. **Write down motion differential equation** of the below mechanical system. *f,* external force, *B,* viscous friction between ground and the cart, *M* is cart mass. a. **Draw free body diagram** and b. write down **force equilibrium equations**.) **(30 puan/points)**



2. a. Aşağıdaki devrenin **dinamik diferansiyel denklem eşitliklerini** yazınız ve b. **E - i_1, and E - i_2 transfer fonksiyonlarını** elde ediniz. (a. Write down **dynamic differential equations of below circuit** and b. obtain **transfer function between E and i_1, and E and i_2.**) **(25 puan/points)**



3. Aşağıdaki **devrenin eşdeğer direnç değerini** sayısal olarak bulunuz. (Calculate the **equivalent resistance of the below circuit**) **(20 puan/points)**



4. Aşağıdaki durum uzay modeli ile verilmiş sistemin, **giriş ve çıkışı arasındaki transfer fonksiyonunu bulunuz**. (find out **transfer function of the below state space model**) **(25 puan/points)**

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-4 & -1\\ 3 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 1\end{bmatrix}u$$

$$y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$\mathcal{L}\left\{\frac{d}{dt}f(t)\right\}=sF(s)-f(0)$$

$$\mathcal{L}\left\{\frac{d^2}{dt^2}f(t)\right\}=s^2F(s)-sf(0)-\dot{f}(0)$$

$$\mathcal{L}\left\{\frac{d^n}{dt^n}f(t)\right\}=s^nF(s)-s^{n-1}f(0)-s^{n-2}\dot{f}(0)-\cdots\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$$

$$A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\quad A^{-1}=\frac{1}{|A|}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$$

$$G(s)=C(sI-A)^{-1}B+D$$

$$R=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

$$R=R_1+R_2+R_3$$

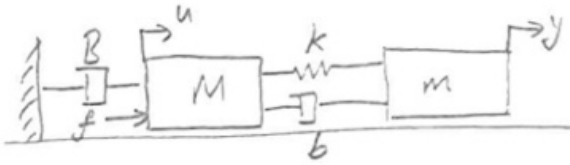
$$R=\frac{e_R}{i}\qquad e_L=L\frac{di_L}{dt}\qquad e_C(t)=\frac{1}{C}\int_0^t i\,dt$$

$$F=kx\qquad F=b(\dot{x}_1-\dot{x}_2)=b\dot{x}\qquad ma=\sum F$$

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1) $f_b = b(\dot{y}-\dot{u})$

$f_B = B\dot{u}$ $\quad f_k = k(y-u)$

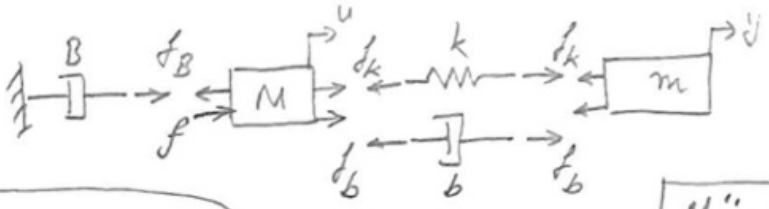


$M\ddot{u} = f - f_B + f_k + f_b$

$m\ddot{y} = -f_k - f_b$

$M\ddot{u} = f - B\dot{u} + k(y-u) + b(\dot{y}-\dot{u})$

$m\ddot{y} = -k(y-u) - b(\dot{y}-\dot{u})$



$$M\ddot{u} + (B+b)\dot{u} + ku = f + b\dot{y} + ky$$

$$m\ddot{y} + b\dot{y} + ky = b\dot{u} + ku$$

---

2) $E = R_1 i_1 + \frac{1}{C}\int (i_1 - i_2)\,dt$

$0 = \frac{1}{C}\int (i_2 - i_1)\,dt + L\frac{di_2}{dt} + R_2 i_2$

$$E(s) = \left(R_1 + \frac{1}{Cs}\right) I_1(s) - \frac{1}{Cs} I_2(s) = \left(\frac{R_1 Cs + 1}{Cs}\right) I_1(s) - \frac{1}{Cs} I_2(s)$$

$$\frac{1}{Cs} I_1(s) = \left(\frac{1}{Cs} + Ls + R_2\right) I_2(s) \Rightarrow I_1(s) = (1 + LCs^2 + R_2 Cs)\, I_2(s)$$

$$I_2(s) = \frac{1}{1 + LCs^2 + R_2 Cs}\, I_1(s)$$

$$E(s) = \left[\frac{R_1 Cs + 1}{Cs} - \frac{1}{Cs(LCs^2 + R_2 Cs + 1)}\right] I_1(s)$$

$$= \frac{(R_1 Cs + 1)Cs(LCs^2 + R_2 Cs + 1) - Cs}{C^2 s^2 (LCs^2 + R_2 Cs + 1)}\, I_1(s)$$

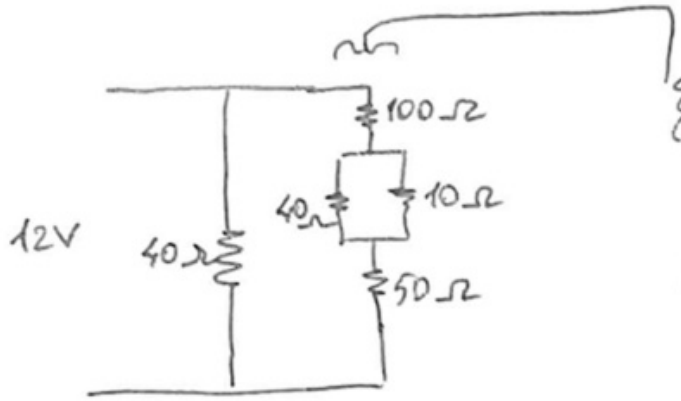
$$\frac{LC^2 R_1 s^3 + R_1 R_2 C^2 s^2 + R_1 Cs + LCs^2 + R_2 Cs + \cancel{1} - \cancel{1}}{Cs(LCs^2 + R_2 Cs + 1)}$$

$$\boxed{\frac{I_1}{E}(s) = \frac{LCs^2 + R_2 Cs + 1}{LCR_1 s^2 + (R_1 R_2 C + L)s + (R_1 + R_2)}}$$

$$E(s) = \left[\left(\frac{R_1 Cs + 1}{Cs}\right)(1 + LCs^2 + R_2 Cs) - \frac{1}{Cs}\right] I_2(s)$$

$$\boxed{\frac{I_2}{E}(s) = \frac{1}{LCR_1 s^2 + (R_1 R_2 C + L)s + (R_1 + R_2)}}$$

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$$\left\{100 + \frac{40.10}{40+10} + 50 = 158\,\Omega\right.$$

$$R_{eq}^{-1} = \frac{1}{158} + \frac{1}{40} \Rightarrow \boxed{R_{eq} = 31.919\,\Omega}$$

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4) $A = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $C = [1 \quad 0]$ $D = 0$

$$G(s) = [1 \quad 0]\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix}\right)^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= [1 \quad 0]\left(\begin{bmatrix} s+4 & 1 \\ -3 & s+1 \end{bmatrix}\right)^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= [1 \quad 0]\left[\begin{pmatrix} s+1 & -1 \\ 3 & s+4 \end{pmatrix} \Big/ s^2+5s+7\right]\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$G(s) = [1 \quad 0]\begin{bmatrix} \frac{s}{s^2+5s+7} \\ \frac{s+7}{s^2+5s+7} \end{bmatrix} = \boxed{\frac{s}{s^2+5s+7} = G(s)}$$

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