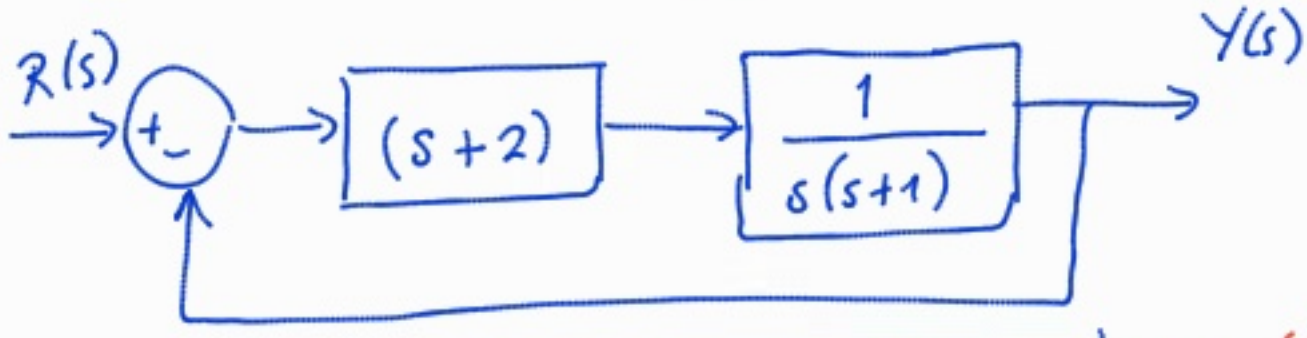
Sur/name:
#

MK43122
Final Exam
1st Question (15 min)

17/06/2021



a) Draw root locus of above control system. (5)
b) Calculate, number of branches, number of asymptotes, asymptote angles, break-in - break-away points. (5)
c) Show all numerical results on the locus. (5)
d) Show the point of the lowest damping ratio this system can get for certain gain value (do not calculate, just show on the locus. (5)

$1 + K\frac{s+2}{s(s+1)} = 0$ char. eq.

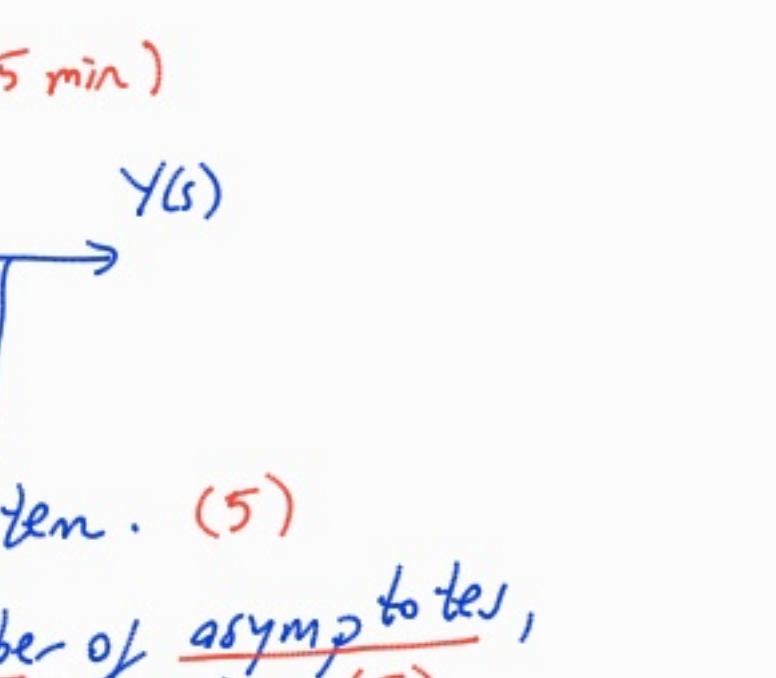
$z_1 = -2$ $m = 1$
$p_1 = 0$, $p_2 = -1$ $n = 2$

$n = 2$ branches
$n - m$ asymptotes

$\theta_1 = \frac{2.0+1}{1} 180^\circ = 180^\circ$ single asymptote

$\frac{d}{ds}\frac{(s+2)/s(s+1)}{} = \frac{1.\,s(s+1) - (s+2)(2s+1)}{[s(s+1)]^2} = 0$

$s^2 + 4s + 2 = 0 \Rightarrow s_1 = -0.6,\ s_2 = -3.4$



$\theta_i = \frac{2i+1}{n-m} \cdot 180^\circ$

$i = 0, 1, \ldots, n-m-1$

$\frac{d G_1(s) H_1(s)}{ds} = 0$

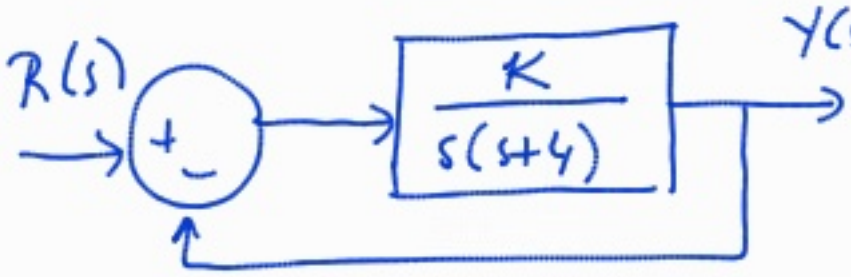
$\zeta = \frac{\sum \text{real part of the poles of } GH - \sum \text{real part of the zeros of } GH}{n-m}$

tonozdayim.com/@Zedeleyici31

Sur/name:
#

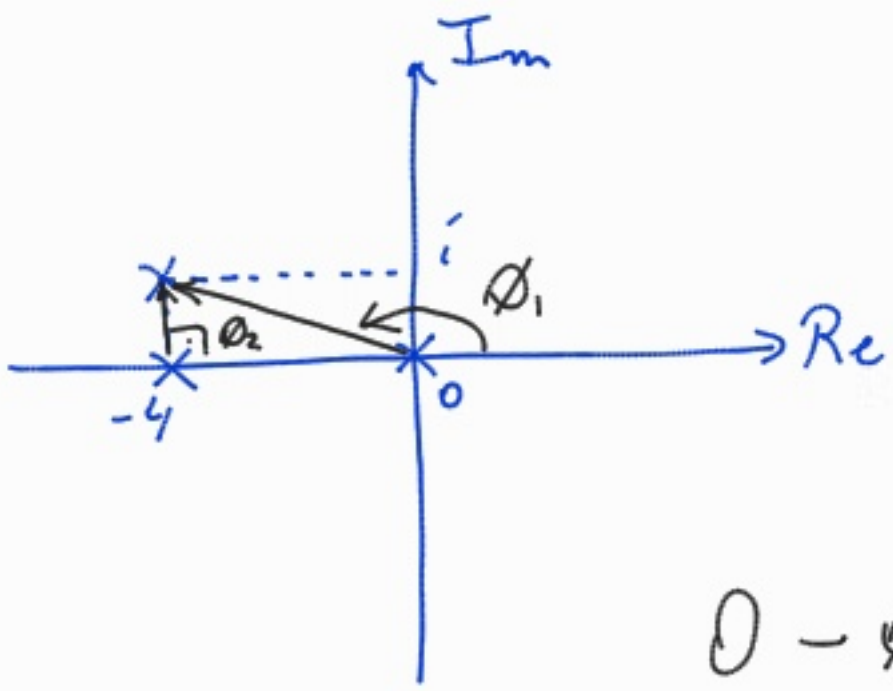
MK 3122
Final Exam
2nd Question (15 min.)

17/06/2021



Control system is given on the left.

- If root locus passes through $s_1 = -4 + i$ for any certain K? (check angle condition) (10)
- how many degrees of contribution is needed by a controller? (10)



$$0 - \phi_1 - \phi_2 = (2k+1)\pi$$

$$-90 - \underbrace{\tan^{-1}\frac{4}{1}}_{75.96} - 90 = (2k+1)\pi$$

$-255.96° \neq -180$ for $k=-1$

$+75.96°$ lead is needed in order to pass intended point.

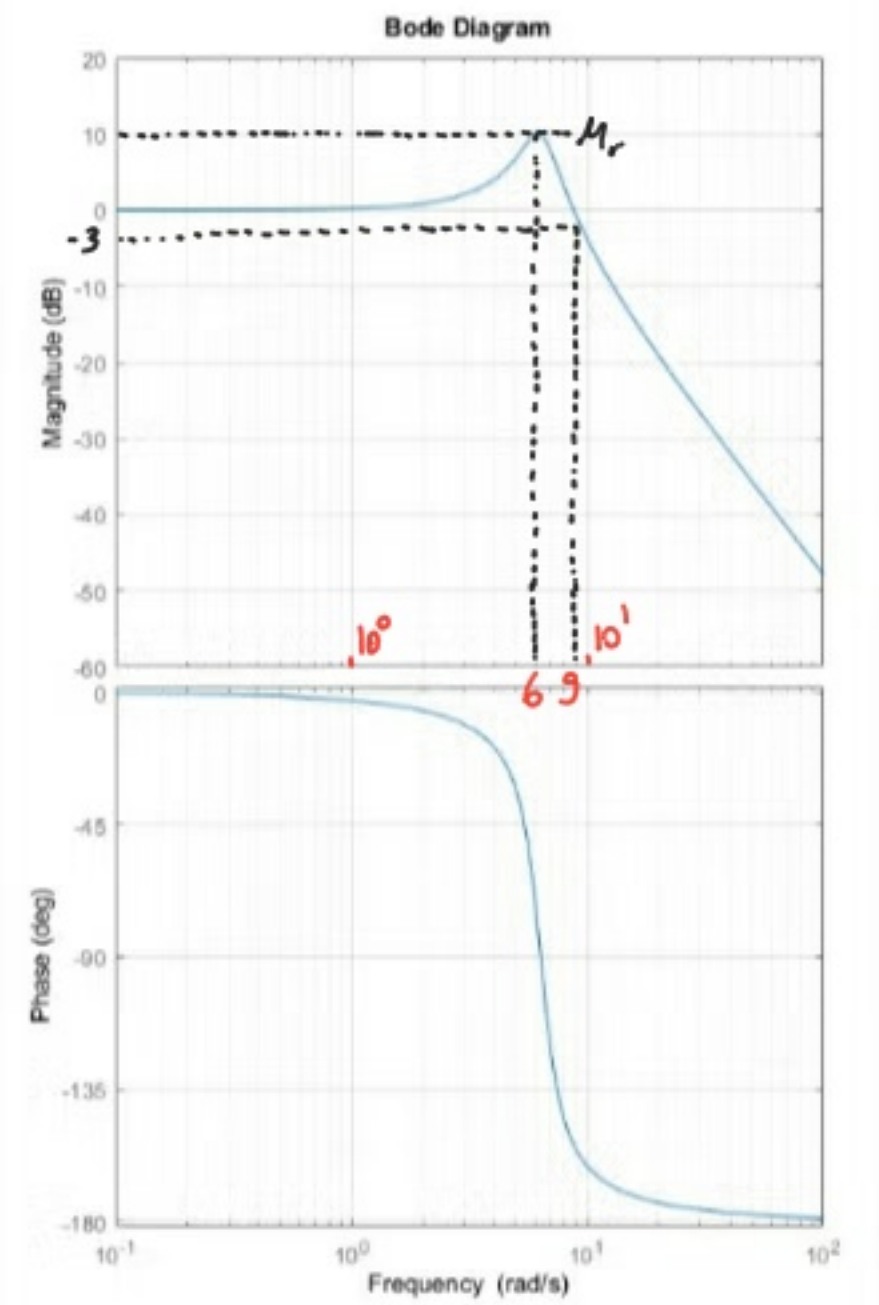
$$\angle G_1(s_1)H_1(s_1) = (2k+1)\pi = \sum_{k=1}^{m} \angle s_1 + z_k - \sum_{j=1}^{n} (s_1 + p_j)$$

tonozdayim.com/@Zedeleyici31

Sur/name:
#

MK 3122
Final Exam
3rd Question (5 min)

17/06/2021



Frequency response of a control system is given on the left. Find out,

- Resonant peak (dB) (5)
- Resonance frequency (rad/sec) (5)
- Bandwidth of the system (5)

Be careful!
(Frequency axis is in log scale)

$M_r = 10\,dB$

$\omega_r = 6\,rad/sec$

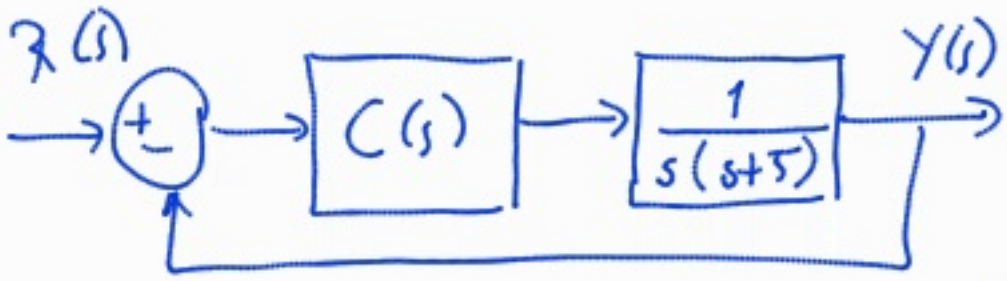
$BW = 9\,rad/sec$

tonozdayim.com/@Zedeleyici31

Sur/name:
#

MK 3122
Final Exam
4th Question (20 min)

17/06/2021



Control system is given on the left.

- If $C(s) = K$ (proportional contro), find out pole locations of the control system in terms of K (roots of characteristic equation) (10)

- If $C(s) = K(s+a)$ (proportional-derivative) control, what are K and a for $\zeta = 0.8$ and $\omega_n = 10$ rad/sec. (10)

$C(s)=K \Rightarrow \frac{Y}{R}(s) = \frac{K}{s^2+5s+K}$, $s_{1,2} = \frac{-5}{2} \mp \frac{\sqrt{25-4K}}{2}$

$C(s) = K(s+a)$

$\frac{Y}{R}(s) = \frac{K(s+a)}{s^2+(5+K)s+Ka}$ → char. eq.

$s^2 + 2\zeta\omega_n s + \omega_n^2$

$2\zeta\omega_n = (5+K)$

$\omega_n^2 = Ka$

$2.(0.8).10 = 5+K \Rightarrow \boxed{K = 11}$

$\omega_n^2 = 100 = Ka \Rightarrow \boxed{a = 9.09}$

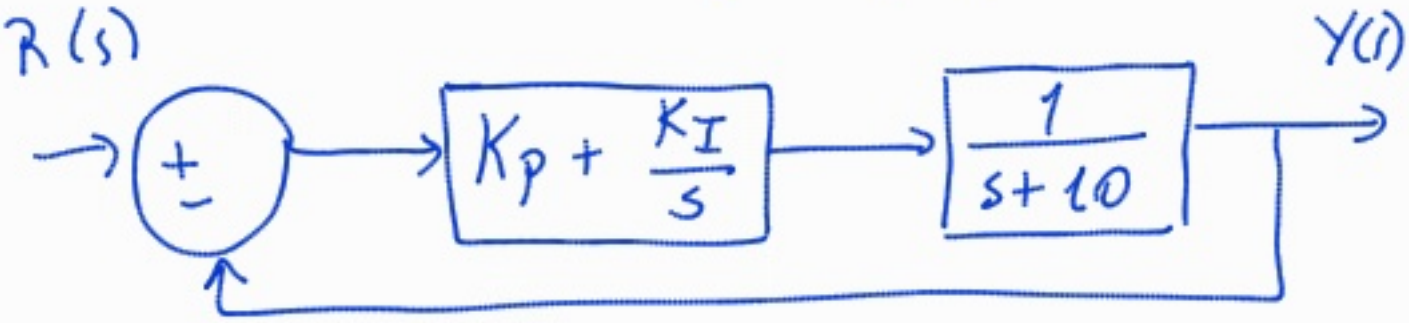
$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$ 2nd order prototype system...

tonozdayim.com/@Zedeleyici31

Surname:
#

MK 3122
Final Exam
5th Question (25 min)

17/06/2021



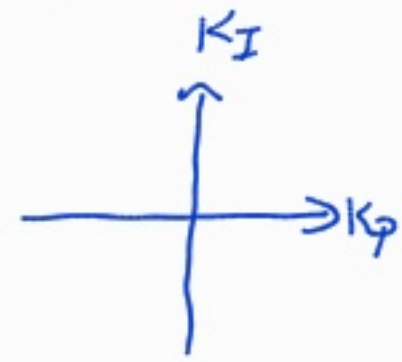
A control system with a proportional integral controller is given above.

(5) Find out transfer function and characteristic equation

(5) " " type number

(5) " " steady state error for unit step/ramp/ parabolic inputs

(10) On K_p-K_I plane, show
- stable region
- $\zeta = 1$ "
- $0 < \zeta < 1$ "
- $K_v = 100$ "



$$G(s) = \frac{K_p s + K_I}{s(s+10)} \Rightarrow \frac{Y}{R}(s) = \frac{K_p s + K_I}{s^2 + (10+K_p)s + K_I}$$

$s(s+10)$ → type 1

$s^2 + (10+K_p)s + K_I$ → char. eq.

type 1 since $K_v = \lim_{s\to 0} sG(s) = K_I/10 \Rightarrow e_{ss} = 10/K_I$ ramp input

$K_p = \lim_{s\to 0} G(s) = \infty \Rightarrow e_{ss} = 1/(1+K_p) = 0$ step input

$K_a = \lim_{s\to 0} s^2 G(s) = 0 \Rightarrow e_{ss} = 1/K_a = \infty$ par. input

$K_p = \lim_{s\to 0} G(s)$ $\quad e_{ss} = \frac{1}{1+K_p}$

$K_v = \lim_{s\to 0} sG(s)$ $\quad e_{ss} = \frac{1}{K_v}$

$K_a = \lim_{s\to 0} s^2 G(s)$ $\quad e_{ss} = 1/K_a$

$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$\zeta = 0$ to type 2nd order system

tonozdayim.com/@Zedeleyici31

Stable region:

$s^2 + (10+K_p)s + K_I$

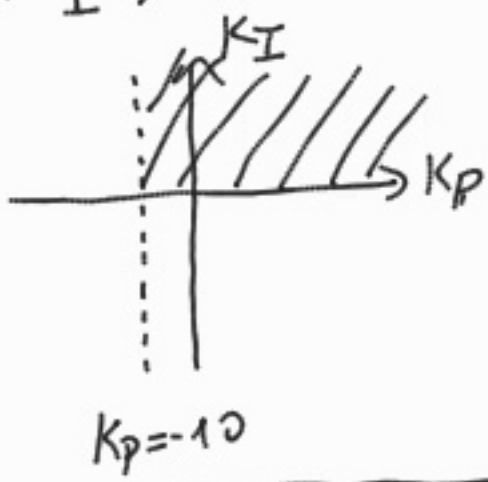
s^2: 1 $\quad K_I$

s^1: $10+K_p$

s^0: K_I

$10+K_p > 0 \Rightarrow K_p > -10$

$K_I > 0$



$\omega_n^2 = K_I$

$\omega_n = \sqrt{K_I}$

$2\zeta\omega_n = 10+K_p$

$\zeta = \frac{10+K_p}{2\sqrt{K_I}} = 1$

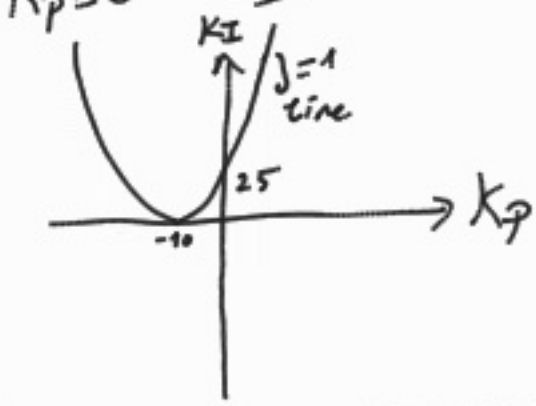
$\frac{10+K_p}{2} = \sqrt{K_I}$

$K_I = \left(5+\frac{K_p}{2}\right)^2$

$= \frac{K_p^2}{4} + 5K_p + 25$

$K_p = -10 \quad K_I = 0$

$K_p = 0 \quad K_I = 25$

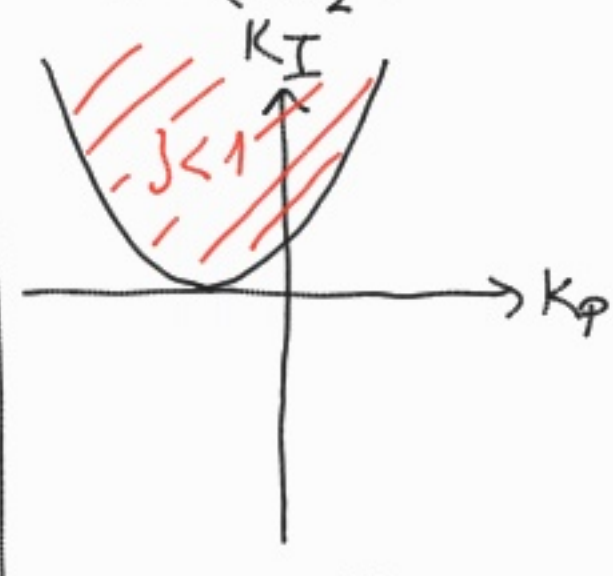


$\zeta = \frac{10+K_p}{2\sqrt{K_I}} < 1$

$(10+K_p) < 2\sqrt{K_I}$

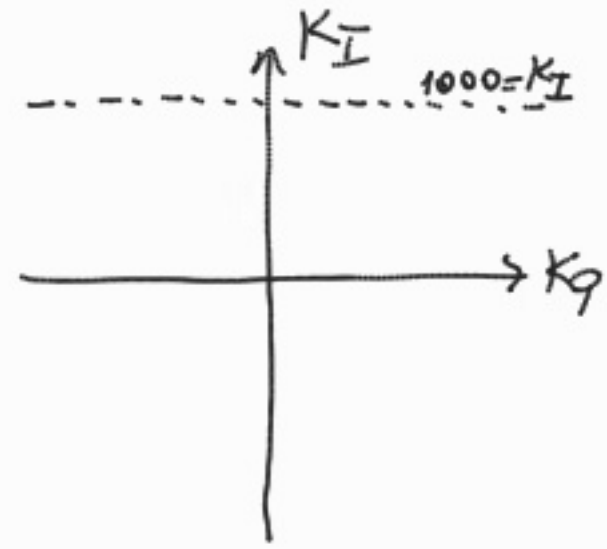
$\left(5+\frac{K_p}{2}\right)^2 < K_I$

$K_I > \left(5+\frac{K_p}{2}\right)^2$



$K_v = \lim_{s\to 0} s\,G(s)$

$= \lim_{s\to 0} \frac{K_p s + K_I}{s+10} = \frac{K_I}{10} = 100 \Rightarrow \boxed{K_I = 1000}$



tonozdayim.com/@Zedeleyici31